

Esercizi su successioni e serie di funzioni

(replicati da [1, 2, 3, 4])

Studiare le proprietà di convergenza delle seguenti successioni di funzioni $\{f_n\}$:

$$f_n(x) = (1 - x)x^n$$

$$f_n(x) = \frac{\sin(nx)}{n}$$

$$f_n(x) = \frac{x^2}{n+x^2}$$

$$f_n(x) = \arctan \frac{x}{n+1}$$

$$f_n(x) = \arctan(nx)$$

$$f_n(x) = e^{-n^2 x^2}$$

$$f_n(x) = \max\{0; 1 - (x - n)^2\}$$

$$f_n(x) = \max\{0; 2 - n(x - 1)\}$$

$$f_n(x) = \left(x^2 + \frac{1}{n^2}\right)^{1/2}$$

$$f_n(x) = \frac{nx}{1+n^2 x^2}$$

$$f_n(x) = \frac{x^n}{n+x^{2n}}$$

$$f_n(x) = \frac{x^{2n}}{1+x^{2n}}$$

$$f_n(x) = \frac{\sin(nx)}{nx} \quad (x \neq 0)$$

$$f_n(x) = n^x$$

$$f_n(x) = \frac{x^n + x^{3n}}{1+x^{2n}}$$

Sia $f_n(x) = nxe^{-nx^2}$. Provare che $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx \neq \int_0^1 \lim_{n \rightarrow \infty} f_n(x) dx$.

Sia $f_n(x) = \frac{\sin(nx)}{n}$ e $f(x) = \lim_{n \rightarrow \infty} f_n(x)$. Provare che $\lim_{n \rightarrow \infty} f'_n(0) \neq f'(0)$.

$$f_n(x) = \sqrt{\sin^2 x + n^{-2}}$$

$$f_n(x) = \frac{3x+n}{x+n} \quad (x \geq 0)$$

$$f_n(x) = n^2(1-x)^n x^2$$

$$f_n(x) = \frac{1+x}{x^n+n^2} \ (x \geq 0)$$

$$f_n(x) = n^{n^x}$$

Studiare le proprietà di convergenza delle seguenti serie di funzioni:

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{x+1}}$$

$$\sum_{n=1}^{\infty} |x|^{nx}$$

$$\sum_{n=1}^{\infty} \frac{|x|^{\sqrt{n}}}{n}$$

$$\sum_{n=1}^{\infty} \frac{x^n}{1+|x|^{n^2}}$$

$$\sum_{n=1}^{\infty} \sin\left(\frac{x}{n}\right)$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} (1 - x^{2n})$$

$$\sum_{n=0}^{\infty} \frac{x^n}{2^n}$$

$$\sum_{n=0}^{\infty} \frac{x^n}{(n+1)2^n}$$

$$\sum_{n=0}^{\infty} \frac{(x+3)^n}{(n+1)2^n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n} x^{2n}}{2n}$$

$$\sum_{n=1}^{\infty} [1 - (-2)^n] x^n$$

$$\sum_{n=0}^{\infty} a^{n^2} x^n \quad (\text{con } a \in (0, 1))$$

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2} x^n$$

$$\sum_{n=1}^{\infty} n^x x^n$$

$$\sum_{n=1}^{\infty} \frac{n^x}{x^n}$$

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{1+x^{2n}}$$

$$\sum_{n=1}^{\infty} x^{\sqrt{n}}$$

$$\sum_{n=1}^{\infty} n^{nx} x^n$$

$$\sum_{n=1}^{\infty} x^{\ln n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^x + (\ln n)^x}$$

$$\sum_{n=1}^{\infty} (2^{3n} + 3^{2n}) x^n$$

$$\sum_{n=1}^{\infty} n^3 x^{2n+1}$$

$$\sum_{n=1}^{\infty} \left(\frac{3n-2}{n+1} \right)^n x^{2n}$$

$$\sum_{n=1}^{\infty} a_n x^n, \text{ con } a_n := \sum_{k=1}^n k^7$$

$$\sum_{n=1}^{\infty} \left(1 - \frac{4n^2+7n+1}{n^3+18n+2} \right)^n x^n$$

$$\sum_{n=1}^{\infty} \left(\frac{x+2}{x^2+1} \right)^n$$

$$\sum_{n=1}^{\infty} \frac{n}{2n+4} e^{nx}$$

$$\sum_{n=1}^{\infty} \left(\sqrt{4n^2 + n} - 2n \right) x^n$$

$$\sum_{n=1}^{\infty} \frac{nx^{n+1}}{(x+1)^n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^{n^2}}{n!}$$

$$\sum_{n=1}^{\infty} x^{2n} \sin(n\pi/4)$$

$$\sum_{n=1}^{\infty} \frac{x^n}{(n+1) \ln(n+1)}$$

$$\sum_{n=1}^{\infty} x(1-x)^n$$

$$\sum_{n=1}^{\infty} x^n(1+x)$$

$$\sum_{n=1}^{\infty} \frac{(1-x^{2n})^{1/3}}{3^n}$$

$$\sum_{n=1}^{\infty} (x \ln x)^n$$

Bibliography

- [1] T.M. Apostol: Calcolo, volume terzo, analisi 2. Bollati Boringhieri 1978.
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- [3] J.P. Cecconi, L.C. Piccininini e G. Stampacchia: Esercizi e problemi di analisi matematica, I vol. Liguori Editore 1996.
- [4] E. Giusti: Analisi matematica 2. Bollati Boringhieri 2003.